CHAPTER I

Water Turbines—General Characteristics

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1. Introduction.

In view of the limited extent of this section on water turbines, the authors of Chaps. I–IV have omitted some of the elementary theory of water turbines, on the assumption that the reader will either already be familiar with it or, alternatively, will be able to obtain access to it in any of the standard works. They are thus able to present more information which will be of value to the practical engineer, directly concerned with the development of hydro-electric power.

Chap. I deals with general characteristics of water turbines and contains some information in § 5 on selection of the type of turbine best suited to the conditions of a particular installation. Chaps. II, III and IV cover respectively the principal types of turbines now in use—Pelton, Francis, Kaplan and propeller—in some detail.

It should be noted that certain of the sub-headings, such as the phenomenon of cavitation and the design of the principal component parts of reaction turbines, are treated in more than one of these chapters and that for the fullest information reference should be made to each of them. These chapters deal with the design, manufacture, and testing of water turbines; for the effect which the type and design of turbines have on the planning and layout of power stations as a whole, the reader should refer to Chap. XVI.

2. Classification of Turbines.

Water turbines are subject to more varied conditions than other prime movers. Heads from 5800 ft. down to 3 ft. and single machines with outputs ranging from fractional horse-power to 200,000 b.h.p. are in use at the present date. The design of water turbines has to cover an almost infinite number of combinations of heads, speeds, and outputs; hence it is necessary to classify them according to the types in general
use, and each type according to the hydraulic conditions for which it
gives the best performance.

The water turbines which are best suited for operating electric
generators have now almost exclusive consideration from the engineer
and belong to three groups:

Pelton wheels, which are impulse turbines.
Francis turbines
Propeller and Kaplan turbines} which are reaction turbines.

The hydraulic characteristics of any water turbine can be sum-
mariesd conveniently by reference to its specific speed, which is a most
useful index to its capabilities, and is represented by the symbol \( n_s \).

The three groups cover a range of \( n_s \) as follows:

(a) Pelton wheels: from 2 to 8 (British units) for single-jet machines, and up
to 11 for double-jet machines.
(b) Francis turbines: from 14 to 90.
(c) Propeller and Kaplan turbines: from 70 to 220.

Before defining \( n_s \), the principle of hydraulic similarity must be con-
considered.

3. Hydraulic Similarity.

Any turbine can be operated under various heads and, at each head,
it will run over a range of speeds; for each head, however, there is one
speed at which it has the best efficiency. For a well-designed turbine,
this speed corresponds to the conditions of smooth entry of water into
the revolving runner and to a minimum of losses throughout.

Let \( H \) be the head in feet and \( n \) the speed in revolutions per minute.
All passages by which the water flows through the runner are best
suited to the directions of the water velocities prevailing at the de-
signed speed of rotation.

If the same turbine is worked under a different head \( H' \), the
geometrical picture of the water velocities will be changed, but at a
certain speed \( n' \) the direction of the water velocities in all parts of the
turbine will coincide with the direction prevailing under head \( H \) at the
speed \( n \). Only the magnitude of the velocities has changed. As the
total energy available has altered from \( H \) to \( H' \), the squares of all the
velocities and the friction losses must have altered in the ratio of the
heads. The velocities have preserved their directions but in magnitude
they have changed in the ratio \( \sqrt{H'}/\sqrt{H} \).

At a given point in the turbine and under the head \( H \), assume the
absolute water velocity to be \( C \), the water velocity relative to the runner
is \( W \) and the peripheral velocity of the runner is \( U \). Under the different head \( H' \), there are at the same point the corresponding water velocities \( C' \) and \( W' \) and the peripheral velocity \( U' \).

Hydraulic similarity prevails when

\[
\frac{C}{C'} = \frac{W}{W'} = \frac{U}{U'} = \sqrt{\frac{H}{H'}}
\]

The speeds of revolution \( n \) and \( n' \) and the discharges \( Q \) and \( Q' \) are in the same ratio as the velocities and, therefore,

\[
\frac{n}{n'} = \frac{Q}{Q'} = \sqrt{\frac{H}{H'}}
\]

The outputs \( N \) and \( N' \) developed at the turbine shaft, being the product of head and discharge, are in the ratio

\[
\frac{N}{N'} = \frac{HQ}{H'Q'} = \frac{H\sqrt{H}}{H'\sqrt{H'}}
\]

Finally, if \( \eta_n \) and \( \eta_n' \) are the hydraulic efficiencies of the turbine under the operating conditions just defined, then \( \eta_n = \eta_n' \) because of congruent angles and similarity of losses.

If two turbines are considered to be geometrically similar and have runners of diameters \( D \) and \( D^* \) respectively, corresponding dimensions are in the ratio

\[
\frac{D}{D^*}
\]

If we assume that these two turbines operate under identical heads, at a certain pair of speeds \( n \) and \( n^* \), the corresponding velocities \( C \) and \( C^* \), \( W \) and \( W^* \), \( U \) and \( U^* \) form identical patterns in direction and magnitude. Because

\[
U = \frac{\pi D n}{60} = U^* = \frac{\pi D^* n^*}{60}
\]

the speeds of revolution are in inverse ratio of the diameters:

\[
\frac{n}{n^*} = \frac{D^*}{D}
\]

The discharges are proportional to the areas of the passages and, therefore, in the ratio

\[
\frac{Q}{Q^*} = \left(\frac{D}{D^*}\right)^2
\]
The horsepowers \( N \) and \( N^* \) are in the ratio

\[
\frac{N}{N^*} = \frac{QH}{Q^*H} = \left(\frac{D}{D^*}\right)^2
\]

and, because of congruent velocities, the efficiencies are identical except for the friction losses which become proportionately smaller for larger diameters (see "Scale Effect", § 8).

4. Specific Speed.

Specific speed is defined as the speed in revolutions per minute at which a turbine would run at the best efficiency for full guide-vane opening under a head of one foot, its dimensions having been adjusted to produce one horse-power.

Consider a turbine of runner diameter \( D \) operating under a head \( H \) (both in feet) and giving at full guide-vane opening the output \( N \) in b.h.p. at the best efficiency and at a speed \( n \) measured in r.p.m.

From § 3 it is clear that, by reducing the head to one foot, the same turbine will operate at its best efficiency when its speed of revolution becomes

\[
n_1 = \frac{n}{\sqrt{H}} = \text{revolutions per minute under 1 ft. head;}
\]

the discharge becomes \( Q_1 = \frac{Q}{\sqrt{H}} \); and

\[
N_1 = \frac{N}{H\sqrt{H}} \text{ is the output under 1 ft. head.}
\]

In order to reduce this power \( N_1 \) to one h.p., the diameter \( D \) of the runner is now altered to \( D^* \), so that

\[
\frac{N_1}{1} = \left(\frac{D}{D^*}\right)^2 \quad \text{or} \quad \frac{D}{D^*} = \sqrt[4]{N_1}
\]

By this change of diameter, the speed is altered in inverse ratio

\[
n^* = \frac{D}{D^*} = \sqrt[4]{N_1}
\]

and the \( n^* \) obtained is the specific speed \( n_s \) by definition. Therefore

\[
n_s = n_1\sqrt{N_1} = \frac{n}{\sqrt{H}} \sqrt{\left(\frac{N}{H\sqrt{H}}\right)}
\]
or, as usually written,

\[ n_s = \frac{n}{H} \sqrt{\left( \frac{N}{\sqrt{H}} \right)} \] or \[ n \frac{\sqrt{N}}{H^{5/4}} \] or \[ n \frac{N^{1/2}}{H^{5/4}} \]

If \( H \) is in feet and \( N \) in British horse-power, \( n_s \) is obtained in British units. Should metric units be used, \( H \) in metres and \( N \) in metric horse-power, the result is \( n_s \) (metric). For conversion, the following relation applies:

\[ n_s \text{ (metric)} = 4.45 \times n_s \text{ (British)} \]

Fig. 1.1.—Classification of water turbines according to specific speed \( n_s \).

The comparative size of the runners can be gauged from the dimensions and the scale. Each runner develops 100 b.h.p. under 40 ft. net head.

The normal running speeds are equal to 10 times \( n_s \) (British).

Fig. 1.1 shows the three types of turbine runners: Pelton, Francis, and Kaplan, placed in their respective positions on a baseline graduated in specific speeds (British). These runners are all drawn to the same scale, and the relative sizes are such that each would produce the same output under the same head. For practical reasons, the output has been taken as 100 b.h.p. for a head of 40 ft. The running speed in r.p.m. works out to \( 10n_s \) as noted in the scale at the bottom of the figure. Fig. 1.1 is intended to convey an appreciation of the very wide range of shapes, sizes and speeds that the turbine designer can offer. Between the seven runners drawn, intermediate sizes and transitions in proportions can be visualized.
For each individual hydro-electric scheme the most suitable type of turbine must be decided upon. This subject is treated in § 5.

5. Choice of Type of Turbine.

The head $H$ in feet under which the turbine will operate gives the first guide to the selection of the type of turbine. Referring to fig. 1.2,

![Graph showing the limits of head for various specific speeds](image)

**Fig.1.2.—Limits of head for various specific speeds**

The field of operation for each type of turbine is limited by its highest and lowest $n_s$, and by the maximum head under which it can safely be operated.

the field of operation of each type is shown in relation to the head $H$ and specific speed $n_s$. By following the horizontal line corresponding to the given head, the types that can be used effectively can be seen at a glance.

As an example, for a head in excess of 2000 ft., the Pelton turbine
offers the only solution. For a head of 400 ft., the Pelton and Francis types can both be used. Again, should the head be 100 ft., the horizontal line crosses the field of all the three types of turbine. Any one of the three could be used so that a further selection according to \( n_s \) becomes necessary.

The total horse-power to be installed must be known and the number of machines then chosen by economic considerations of load factor, extent of water storage if any, cost of power house, convenience of operation and maintenance. Once the output per machine has been decided, information must be obtained concerning the suitable speeds for which the generator can be constructed economically. From these data:

\[
N = \text{turbine output in horse-power at full load}, \\
H = \text{the effective head in feet}, \\
n = \text{normal running speed in r.p.m.},
\]

the specific speed is calculated as

\[
n_s = \frac{n}{H} \sqrt{\left(\frac{N}{\sqrt{H}}\right)}
\]

The appropriate specific speed is indicated in fig. 1.2, which shows the upper limit of head for which each particular \( n_s \) is suitable for each type of turbine. Should the head exceed this permissible limit, a lower speed \( n \) or a lower output \( N \) must be adopted in order to reduce the value of \( n_s \).

The range of specific speeds will be noted:

Pelton turbines with one jet extend up to \( n_s = 8 \) (British).

Pelton turbines with two jets extend up to \( 8\sqrt{2} = n_s = 11 \) (British).

Pelton turbines with four jets extend up to \( 8\sqrt{4} = n_s = 16 \) (British).

The Francis turbine range from \( n_s = 14 \) to \( n_s = 90 \) overlaps the Pelton range (four jets) and the propeller and Kaplan range with \( n_s \) over 70.

Since the speed of the generator can generally be selected for several suitable numbers of pole pairs, the appropriate specific speed is not limited to one value only. The overlap is thus considerably extended and, in many cases, the problem arises of selecting from two types of turbine, either of which could be used.

Here a wider knowledge of the advantages and disadvantages of each type will assist, especially with respect to efficiency when running at part load. This is shown in fig. 1.3, where efficiency relative to output is illustrated for some typical examples.

Should the machines be called upon to operate for long periods at
small loads, the Pelton turbine would have preference over the Francis turbine. Similarly, if the choice lay between two Francis turbines, that with the lower specific speed would be the more suitable, whereas, if the choice lay between a Kaplan turbine and a propeller turbine or a Francis turbine with a high specific speed, the Kaplan type would be preferred. This is because the flattest efficiency curve is obtained from the Kaplan turbine, followed by the Pelton turbine, the low-specific-speed Francis turbine, the higher-specific-speed Francis turbine, and finally the propeller turbine which has the most peaked form of efficiency curve.

![Graph showing efficiency at various loads](image)

**Fig. 1.2—Efficiencies at various loads**

The efficiencies obtainable at partial guide-vane opening differ considerably according to the specific speed $n_s$.

Generally, a cheaper machine and a less costly power house result from a higher speed of revolution.

Fig. 1.2 requires some further explanation. The upper limits for the heads given are obtained by computation from existing plants. They are not to be regarded as absolute and final but indicate that turbines have been built for and are in successful operation at heads below this line. The limit is, therefore, subject to revision as further experience with new plant becomes available.

It must not be assumed that the highest possible specific speed is
always desirable, and the choice of a value of $n_s$ below the limit line does not mean that the plant is not modern. Again, if the value of $n_s$ is chosen on the limit or very near to it, it is advisable to inquire into the behaviour of existing similar machines, because the limit line is no guarantee of perfect performance; experience concerning newly installed plant is not readily available until some years of operation have passed.

Apart from statistical information from existing installations, the limitations to $n_s$ fall under various headings:

(a) **Hydraulic Limitations.**

The curve shows a limit of $n_s = 8$ for single-jet Pelton turbines (also $n_s = 11$ for two jets, $n_s = 16$ for four jets). This limit is due to space not permitting a larger jet diameter in relation to the pitch-circle diameter of the runner. This ratio of jet diameter to runner (pitch-circle) diameter fixes the value of $n_s$.

(b) **Mechanical Strength.**

In § 9 it is shown that any one type of turbine of given $n_s$, characterized by all its geometrical proportions, is subjected to stresses that are proportional to the head under which it operates. In deciding on the use of a given $n_s$ for a head higher than that of the prototype, the designer faces new problems of mechanical strength (and bearing-surface pressures). Sometimes by changing the materials used, higher stresses can be allowed, but more often the proportions of the turbine must be modified, for instance by increasing the thicknesses. This causes some departure from true geometrical similarity and may affect performance.

(c) **Best Maximum Efficiency.**

Experience shows that each type of turbine, Pelton, Francis, propeller and Kaplan, has a best maximum efficiency over a narrower field than generally shown in fig. 1.3. For instance, impulse wheels may give best efficiencies at very high heads that may be inferior to those at lower heads. The very large values of $n_s$ attainable with Kaplan and propeller turbines depend on high recovery in the draft tube of the velocity head, and extremely high specific speeds are obtainable only at some sacrifice of efficiency. It is not to be inferred that this is bad practice; there are notable instances of flood conditions where maximum output at low heads is of paramount importance. In such cases the Kaplan turbine with its generous overload capacity is at its best, and it does not matter greatly if the efficiency percentage drops a little, so long as the required output is available.

(d) **Reynolds' Number.**

Reynolds' number provides a basis for comparing machines, according to the nature of the flow through them, when they are geometrically similar but are of different sizes, as defined by the diameter $D$, and operate under various heads which affect the velocity $v$ of the water at any given point.

The Reynolds number is

$$R = \frac{vD}{\nu}$$
where $v$ is the kinematic viscosity of water, which can be considered constant because variations in temperature are small. Since $v$ is proportional to $\sqrt{H}$, one and the same runner operates at increasing Reynolds numbers $R$ as the head increases, maintaining hydraulic similarity.

Since no information from actual plant exists beyond the limits given in fig. 1.2, there is no assurance that the nature of eddy formation will remain acceptable above such heads. In other words, there is a possibility of unexpected vibrations if these limits of head are exceeded. The size of the unit has a similar effect. For one and the same head, turbines of identical $n_s$ but of larger diameters than any actually in operation may be required to work at higher Reynolds numbers and may then show unexpected vibrations because of eddies. The information on this aspect available at present is very inadequate. Statistical methods are difficult to apply because details of design vary widely and are seldom divulged. Therefore, it is wise to be cautious and conservative. The influence of an increasing Reynolds number is found in the scale effect which is treated in § 8.

(e) Setting.

As mentioned in § 6, in order to ensure satisfactory operation, reaction turbines of given $n_s$ require a lower setting of the turbine in relation to the tailwater level when the head $H$ increases. This may result in such unusually deep excavation that the cost of it may well absorb any saving on the machines arising from a higher speed.

(f) Generator.

For very large outputs, the generator designers, having particular regard to safety at maximum runaway speed, may fix a limit to the permissible operating speed. The ratio of maximum runaway speed to optimum running speed $n$ is ascertained from model turbines tested in the laboratory, and generally this ratio increases with the specific speed $n_s$.

6. Turbine Setting and Cavitation.

Impulse turbines, i.e. Pelton wheels, must operate well clear of the tailwater to ensure proper ventilation. Should the tailrace water be subject to large variations in level, the Pelton turbine must be placed sufficiently high to clear it at all times. An appreciable head may thus be lost permanently, and this is of considerable importance if the tailwater level should be subject to high floods or tides.

In such cases, preference is given to the Francis turbine, where the reverse prevails, i.e. the runner must be placed lower than a certain permissible level relative to the minimum tailwater level. This applies to all reaction turbines. If they are placed too high, their operation is affected by the phenomenon of cavitation, resulting in pitting and erosion of the runner blades to various degrees, and also noise and vibrations. In the most severe cases this may result in gradual destruction by fatigue due to repeated hammering.

Each type of reaction turbine has its own susceptibility to cavit-
ation and the following considerations will define the limits allowable for the static suction \( H_s \) (fig. 1.4).

\( H_s \) represents the elevation in feet of the runner above the minimum tailwater level for one turbine operating at full load. The velocity of water at discharge from the runner is \( C_2 \) and that at the exit of the draft tube \( C_4 \), both in feet per second. Assuming purely axial discharge

![Fig. 1.4.—Setting of a reaction turbine](image)

In order to avoid cavitation, the elevation \( H_s \) of the runner above minimum tailwater level must not be allowed to exceed

\[
H_s = H_B - \sigma H.
\]

Values of \( \sigma \) for each particular \( n_3 \) are given in fig. 1.5.

\( H_B \) is obtained from fig. 1.6.

and even distribution of velocity at entrance and exit of draft tube, the velocities \( C_2 \) and \( C_4 \) are in inverse ratio to the cross-sectional areas of the water passages at the corresponding places.

The equation of energy for these two sections is

\[
\frac{p_2}{\gamma} + \frac{KW_2^2}{2g} + \frac{C_2^2}{2g} + H_s = \frac{C_4^2}{2g} + \frac{\sum h_2}{2} + H_a \quad \ldots \quad (1)
\]

where \( \sum h_2 \) is the sum of losses from section 2 to section 4 in feet head of water,

\( H_a \) is the atmospheric pressure over the tailrace in feet of water,

\( \frac{p_2}{\gamma} \) is the absolute pressure at point 2, in feet of water,

\( \frac{KW_2^2}{2g} \) is the local depression at the back of the runner vanes, caused by the distribution of velocity \( W_2 \) peculiar to each type of turbine. \( K \) is a suitable coefficient.
Cavitation is avoided as long as the absolute pressure \( \frac{p_2}{\gamma} \) remains well above the vapour pressure of the water. If the height of the barometric water column in feet is \( H_B \), the condition for cavitation-free operation is \( \frac{p_2}{\gamma} > H_a - H_B \) which can be written from (1)

\[
\frac{p_2}{\gamma} = H_a + \frac{C_4^2}{2y} + \frac{4}{2} \sum h_z - \frac{KW_2^2}{2y} - \frac{C_2^2}{2y} - H_s > H_a - H_B
\]

If \( \eta_n \) is the recovery factor of the velocity head at runner exit, then

\[
\eta_D \frac{C_2^2}{2y} = \frac{C_2^2}{2y} - \frac{C_4^2}{2y} - \frac{4}{2} \sum h_z
\]

and the previous inequality becomes

\[
H_B - H_s > \eta_D \frac{C_2^2}{2y} + \frac{KW_2^2}{2y}
\]

Supposing it is observed from the behaviour of a particular turbine that cavitation begins at a certain critical value of suction head \( H_s \) which will be designated by \( H_c \); it is proposed to calculate the corresponding critical value \( H_c' \) when the same turbine operates under a different head \( H' \). The comparison must be based on conditions of hydraulic similarity. Therefore, all velocity heads and friction losses are in proportion to the operating head \( H \). By dividing both sides of the inequality by \( H \), then

\[
\frac{H_B - H_c}{H} = \eta_D \frac{C_2^2}{2gH} + \frac{KW_2^2}{2gH}
\]

Because of similarity of hydraulic conditions,

\[
\frac{C_2^2}{H} = \frac{C_2'^2}{H'} \quad \text{and} \quad \frac{W_2^2}{H} = \frac{W_2'^2}{H'} \quad \eta_D = \eta_D' \quad K = K'.
\]

The right side of the equation \( \eta_D \frac{C_2^2}{2gH} + \frac{KW_2^2}{2gH} \) remains the same in changing from the head \( H \) to the head \( H' \). It can therefore be considered as a measure of the susceptibility of this particular turbine to cavitation, and be called its "critical sigma" \( \sigma_c \), a coefficient independent of the head. Observations of the critical suction \( H_c \) under the head \( H \) permit determination of it as

\[
\sigma_c = \frac{H_B - H_c}{H}
\]
and again for the head $H'$ and the barometric pressure $H_B'$,

$$ \sigma_c = \frac{H_B' - H'_c}{H'} $$

from which

$$ H'_c = H_B' - \sigma_c H' $$

This particular turbine will be free from cavitation as long as the static suction $H'_c$ remains less than the critical static suction $H'_c$. Therefore, for any head it is necessary to ensure that

$$ H_s < H_B - \sigma_c H $$

The larger the susceptibility coefficient $\sigma_c$, the more readily will the turbine cavitate and, to counter this, the lower must the turbine be set in relation to the tailwater level; $\sigma_c$ is a pure number which characterizes each particular turbine. Any turbine that is geometrically similar and operates under similar hydraulic conditions has the same value of $\sigma_c$. With a different design or proportions $\sigma_c$ will generally be different. As can be expected, the specific speed $n_s$ has a great influence on the critical sigma, as is seen from the following considerations:

As given in Chap. III, § 3, the exit diameter $D_2$ of all reaction turbines is related to the speed and discharge by

$$ \frac{D_2^3 n}{Q} \approx \text{constant for all specific speeds} $$

From this it is possible to calculate

$$ \eta D \frac{C_2^2}{2gH} \approx 3.6 \times 10^{-4} (n_s)^{4/3} = \text{dynamic suction} $$

Fig. 1.5 shows this function of $n_s$. From experimental data, safe values of $KW_s^2/(2gH)$ follow the law

$$ K \frac{W^2}{2gH} \approx 4.0 \times 10^{-8} (n_s)^{7/3} = \text{depression at back of vane} $$

Values of this quantity have been shown in Fig. 1.5 in order to give the curve of permissible $\sigma$. It can be seen how rapidly $\sigma$ increases with $n_s$, and thus correspondingly its importance at large specific speeds.

It must always be remembered that the coefficient $K$ depends to a great extent upon each particular design of turbine, namely, the specific loading on the runner blades, their curvatures, and the ratio of chord to pitch.
Some information on this aspect, as related to Francis-runner design, will be found in Chap. III, § 3.

For turbines already installed, from which data concerning cavitating behaviour are available, the actual values of the suction head are recorded. The value

$$\sigma = \frac{H_B - H_s}{H}$$

is then calculated and can be plotted against \( n_s \) as in fig. 1.5. It will be seen that the curve of \( \sigma \) given there represents actual installation values for the lowest tailwater levels with some small margin of safety over the "critical sigma" \( \sigma_c \).

![Fig. 1.5.—Sigma as a function of specific speed](image)

The values of \( \sigma \) shown correspond to safe operation for well-designed turbines.

The experimental determination of \( \sigma_c \) is carried out with small scale-model turbines in the laboratory. The point at which cavitating starts is determined from visual observation of the runner and by the discontinuity that occurs in the performance under conditions of hydraulic similarity when related to the value of \( (H_B - H_s)/H \), which is varied by altering either \( H_s \) or \( H \) or both. By plotting the efficiency, discharge and output under unit head at constant unit speed, against \( (H_B - H_s)/H \), the point of discontinuity is made apparent. This
determines the value of the critical sigma $\sigma_c$. In order to facilitate the calculation of the critical suction head $H_s$ for each particular installation, fig. 1.6 is given where the height of the water barometer in feet is plotted against the altitude in feet above sea-level, and, for various temperatures of water, $H_B$ can be read off directly as a function of these two variables.

![Graph](image)

**Fig. 1.6.—The height of barometric water column $H_B$ depends upon the atmospheric pressure $H_n$ and the temperature. It can be read off with sufficient accuracy in terms of the altitude above sea-level and the temperature of the water.**


Test data that can be obtained from field-testing of turbines are of necessity very restricted. Extensive tests on large installations are costly and usually the time available is very short, because the plant is required in service at the earliest possible date for financial and operational reasons. There are also hydraulic limitations: the head may not be varied, the speed generally is constant and the load available may not be as steady as is desirable for accurate observations.

In order to obtain complete information regarding the characteristics and performance of a water turbine, it is usual to carry out full laboratory tests. A complete replica at a reduced scale is run under a small head. The head is not reduced for the purpose of maintaining the scale, but in order to keep down the rotational speed and the output. As the head is produced artificially by pumping, the consumption of power is important in the economics of the laboratory.
The tests are applied to all types of turbines. Because of their wide
disparity, it is essential to use a higher head for tests of impulse turbines,
as otherwise the proportions become impracticable. It is generally
desirable to arrange for test turbines to be of such dimensions that the
turbine output is not less than 5 b.h.p. or more than, say, 50 b.h.p.
Too small an output may necessitate excessively fine technique for
measurements.

In order to preserve the exact geometrical similitude to the full-size
machines, too small a scale must be avoided. This would lead to
stringent requirements for accuracy in production of the model and
would demand such specialized workmanship as could not be obtained
with the usual production standards of the turbine makers.

Because of these considerations, a model-turbine size is chosen
where the jet diameter for impulse runners is 1–2 in. at normal full load;
while for reaction turbines the runner diameter may be 1 2 ft.

As mentioned, the head is provided by a motor-driven pump; the
discharge is measured, either by a calibrated weir or by the volumetric
method. The output is absorbed in a Prony brake, which is found the
best for work at extremely varied speeds and torques.

The tests consist in running the turbine for a given position of the
spear or guide-vane apparatus from standstill to maximum runaway,
and measuring for each speed, when steady conditions prevail, the data
from which the efficiency is calculated. The test figures of discharge and
speed are reduced to unit head and the results plotted in curves of
efficiency against speed and discharge against speed.

After covering the complete range of spear or guide-vane openings,
the general characteristic curves are worked out. Examples for Pelton,
Francis, propeller, and Kaplan turbines respectively are illustrated in
figs. 1.7–1.10.

The abscissae are the speeds in r.p.m. reduced to unit head,

\[ n_1 = \frac{n}{\sqrt{H}} \]

The ordinates are the discharges reduced to unit head,

\[ Q_1 = \frac{Q}{\sqrt{H}} \]

For each guide-vane opening (or spear opening), a curve gives the dis-
charge \( Q_1 \) versus speed \( n_1 \). The curves of efficiencies are then drawn by
joining together the points of equal efficiency over the whole range of
discharges. These general characteristic curves are sometimes called
"niveau curves" from the resemblance of the lines of constant efficiency to contour lines on a map; a more common term is "mussel curves" because of their likeness to the lines of growth of the shell-fish of that name.

For turbines intended to drive synchronous generators at a nearly constant frequency, a fixed speed of revolution is required and this is represented by the ordinate at the given \( n_1 \) for each prevailing head. A change in head is reflected by a change in \( n_1 \), and therefore for one and the same speed of revolution different heads are represented by different ordinates.

The field of all the discharges versus speeds that can occur is limited on the right-hand side by the runaway curve, which is the line of zero

Fig. 1.7.—General characteristic curves of a model impulse turbine (Pelton wheel) 
\[
\eta = \frac{Q_1}{n_1}
\]

Where:
- \( Q_1 \) = discharge under unit head.
- \( n_1 \) = revolutions per minute under unit head.
- \( \eta \) = overall efficiency of turbine.
- \( s \) = spear stroke as fraction of full stroke.
Fig. 1.8.—General characteristic curves of a model Francis turbine of medium specific speed \( n_s = 50 \)

\( Q_1 \) = discharge under unit head.

\( n_r \) = revolutions per minute under unit head.

\( \eta \) = overall efficiency of model turbine, replica of the full-size turbine from spiral casing inlet to draft tube outlet.

\( \sigma \) = gate apparatus opening as fraction of full opening.
efficiency. This runaway of the model test turbine takes place when the turbine is freed from all external load. For designing the generator, the runaway which is of interest is that of the whole rotating element, turbine runner and generator rotor. This is lower than the runaway of turbine alone, because of the considerable windage and other friction losses of the generator at this abnormally high speed.

Fig. 1.9.—General characteristic curves of a model fixed-blade propeller turbine of \( \eta_s = 90 \)

\( Q_1 \) = discharge under unit head.
\( n_1 \) = revolutions per minute under unit head.
\( \eta \) = overall efficiency of model turbine, replica of full-size turbine from spiral casing inlet to draft tube outlet.
\( a_s \) = gate apparatus opening as fraction of full opening.

The characteristic curves of a Kaplan turbine shown in fig. 1.10 are compiled from a series of curves (fig. 1.9) each corresponding to a different runner-blade opening. The curves of equal efficiencies here are the envelopes of the efficiency curves for each runner-opening superimposed, and are valid only for the optimum combination of runner-blade/guide-vane openings.

The absolute maximum runaway curve arises from a combination which differs from that suitable for maximum efficiency. In difficult
Fig. 1.10.—General characteristic curves of a model Kaplan turbine of $n_2 = 120$

$Q_1$ = discharge under unit head.

$n_1$ = revolutions per minute under unit head.

$\eta$ = overall efficiency of model turbine, replica of full-size turbine from spiral casing inlet to draft tube outlet.

$\alpha$ = Guide-vane opening angle in degrees.

$\beta$ = Runner-vane opening angle in degrees.

The efficiency curves shown correspond to the best combination of guide-vane and runner-vane openings.

cases, judicious limitation of maximum guide-vane openings can reduce the runaway speed very substantially, and thus lighten the task of the generator designer.

8. Scale Effect.

The model test turbines are scaled-down replicas of the full-size machines. The test results from the model turbine are used to ascertain
the anticipated full-size turbine efficiency when working under conditions of strict hydraulic similarity.

As a first approximation, the efficiencies may be considered identical, the losses in each case being proportional to the velocity heads. It has, however, been recognized for many years that, with increase in size, reaction turbines give efficiencies greater than those of the model. This is due to the scale effect.

As long as the full-size turbine had a diameter no more than two or three times that of the model, the scale effect gave to the turbine builder just the margin which he wanted over the efficiency he had guaranteed. Furthermore, water measurements in the field are with reason considered subject to greater inaccuracies than in the laboratory. The tolerance of 2 per cent in the guaranteed efficiency was then of the same order as the scale effect.

With further increase in size, however, the ratio between full-size and model diameters became so large as to make the scale effect of importance in comparing competitive offers from various turbine builders. It therefore became necessary to be able to forecast with sufficient accuracy, by taking into account the scale effect, the efficiencies obtainable in the field.

As the field test, which is seldom accurate within more than one per cent, provides the only experimental basis for determination of the scale effect relative to the model turbine test, the calculation of scale effect is a controversial subject.

If \( \eta \) = the efficiency of the prototype turbine,
\( \eta' \) = the efficiency of the model turbine which is geometrically similar to it in every dimension, inclusive of the clearances at the water seals and the relative roughness of all surfaces,
\( D, D' \) = the respective diameters,
\( H, H' \) = the respective heads,

the losses are 1 \( \eta \) and 1 \( \eta' \) respectively. The problem is to determine the ratio of losses

\[
\frac{1 - \eta}{1 - \eta'}
\]

from which the efficiency \( \eta \) could be calculated, knowing the efficiency \( \eta' \) of the model.

One of a number of solutions offered by Moody is to assume the ratio
of losses to be dependent solely on the diameter ratio
\[
\frac{D'}{D} \quad \text{and of the form} \quad \frac{1 - \eta}{1 - \eta'} = \left(\frac{D'}{D}\right)^n
\]
The exponent \(n\) was then derived from a number of field and model data and was found to be \(n = 0.25\); but \(n = 0.20\) is recommended by the authors as the more conservative figure.
\[
\frac{1 - \eta}{1 - \eta'} = \left(\frac{D'}{D}\right)^{1/5} = \text{Moody scale effect.}
\]
Since, however, the friction losses in pipes are known to depend on the Reynolds number, it is better to consider by analogy that the friction losses in the turbine runner are a function of the Reynolds numbers
\[
\bar{R} = \frac{Dv}{\nu} \quad \text{and} \quad R' = \frac{D'v'}{\nu'}
\]
for the model turbine.

By assuming the same kinematic viscosity \(\nu\) in both cases and since the two turbines must be operating under similar hydraulic conditions, we have
\[
\frac{R'}{\bar{R}} = \frac{D'\sqrt{H'}}{D\sqrt{H}}
\]
Thus the ratio of losses \(1 - \eta\) and \(1 - \eta'\) must be a function of the diameters and of the heads.

Ackeret offers the following solution† which is based, not on overall efficiencies, but on the hydraulic efficiencies of the turbine \(\eta_h\) and \(\eta'_h\) respectively:
\[
\frac{1 - \eta_h}{1 - \eta'_h} = \frac{1}{2} \left[ 1 + \left(\frac{D'}{D}\right)^{1/5} \left(\frac{H'}{H}\right)^{1/6} \right]
\]
The hydraulic losses \(1 - \eta_h\) and \(1 - \eta'_h\) differ from the overall losses \(1 - \eta\) and \(1 - \eta'\) by excluding the bearing and stuffing-box frictions, windage, etc. The difference may be of importance because the similarity between the model turbine and the prototype usually does not extend to bearings, shafts, stuffing boxes, and generally purely mechanical devices.

The Moody formula assumes that these losses are sufficiently similar to make the distinction between hydraulic efficiency and overall

efficiency an unnecessary refinement for the practical determination of the scale effect.

The Ackeret formula is developed on the assumption that the hydraulic losses derive from two causes:

(a) Kinetic losses which remain unaffected by the scale,
(b) Friction losses which vary with the Reynolds number.

The scale effect from Moody or Ackeret is strictly applicable to the point of best efficiency, which is the most important one for practical purposes. The scale effect at other points has not been investigated. It is generally assumed that the step up in efficiency $\eta'$ to $\eta$ calculated for the point of best efficiency is applicable at all guide-vane openings and speeds.

In consequence of the increase in efficiency $\eta'$ to $\eta$, the hydraulic conditions are as though the effective head on the turbine were altered in the ratio $\eta/\eta'$ and, therefore, all velocities and discharges are subjected to an adjustment of $\sqrt{(\eta/\eta')}$ whilst the outputs are affected in the ratio $(\eta/\eta')^{3/2}$.

For impulse turbines, no scale effect is observed. Probably this is due to the deterioration in smoothness of the jet when the head increases—deterioration which nullifies any benefit that could be expected from reduced friction losses.

9. Mechanical Similarity.

As shown in § 3, two identical turbines under heads $H$ and $H'$ operate under similar hydraulic conditions when their speeds $n$ and $n'$ are in the ratio

$$\frac{n}{n'} = \sqrt{\frac{H}{H'}}$$

The hydraulic pressure over any given part of the turbine produces a force proportional to the head.

If $P$ is the force resulting from hydraulic pressure over a surface $F$ under the head $H$, and $P'$ the corresponding force caused by the head $H'$ over the same surface $F$, then

$$\frac{P}{P'} = \frac{H}{H'}$$

The centrifugal forces are proportional to the square of the speed, namely $(n/n')^2$ which is equal to $H/H'$. Thus, the centrifugal forces are in the same ratio as the hydraulic forces and, since the two runners are of
identical size, all stresses are proportional to the heads under which the turbines operate. The same applies to the specific pressures in bearings.

Consider now two turbines geometrically similar, therefore of same specific speed, but of different diameters $D$ and $D^*$ operating under the same head. By definition, these machines are geometrically similar in all their parts. For similar hydraulic conditions their speeds are in inverse ratio of the diameters. Therefore $n^*/n = D/D^*$.

The hydraulic forces $P$ and $P^*$ acting over the geometrically similar areas $F$ and $F^*$ are in the ratio

$$\frac{P^*}{P} = \frac{F^*}{F} = \left(\frac{D^*}{D}\right)^2$$

Centrifugal forces are the products (mass $\times$ radius $\times \omega^2$). Provided the material from which they are made is of the same specific weight, similar parts of the turbines of diameters $D$ and $D^*$ are subjected to centrifugal forces $C$ and $C^*$ in the ratio

$$\frac{C^*}{C} = \left(\frac{D^*}{D}\right)^3 \cdot \frac{D^*}{D} \cdot \left(\frac{n^*}{n}\right)^2 \text{ which reduces to } \frac{C^*}{C} = \left(\frac{D^*}{D}\right)^2$$

The centrifugal forces are therefore in the same proportion as the hydraulic forces $C^*/C = P^*/P = F^*/F$.

The stresses are the quotient of the sum of these forces divided by the areas over which they apply, namely

$$\text{stress} = \frac{P}{F} + \frac{C}{F} \text{ and } (\text{stress})^* = \frac{P^*}{F^*} + \frac{C^*}{F^*}$$

Since

$$\frac{P^*}{F^*} = \frac{P}{F} \text{ and } \frac{C^*}{F^*} = \frac{C}{F}$$

it follows that the stresses are the same in both cases. Hence turbines of geometrically similar design are equally stressed when operating under the same head, irrespective of their size, so long as they operate under similar hydraulic conditions.

It follows that the stresses in turbines geometrically similar and made of the same materials are proportional to the head under which they operate, irrespective of their size, when they operate under similar hydraulic conditions.