Determining corresponding points on Kaplan turbine efficiency diagrams

By Árpad Fáy*

New formulae are developed for the calculation of corresponding points when scaling up efficiency and cavitation parameters. A remarkable extension of Hutton's well-known loss analysis was presented in the March, 1967, issue of WATER POWER in an article by Osterwalder entitled "Model testing for Kaplan-turbine design, including studies on efficiency scale effects". Based on the original and extended theory, an attempt is made in this article to derive more reasonable formulae for the calculation of corresponding points

IN SCALING-UP the efficiency and cavitation parameters from those measured on a model turbine to the full-scale machine, corresponding points are to be determined on the efficiency diagrams of both the prototype and the model concerned. Corresponding points are calculated, furthermore, when the diagrams of test results for both model and prototype are available and scale effects are determined by comparing these diagrams.

The problem of how to determine the corresponding points is discussed here only in cases using $n_{11}-Q_{11}-\eta$ diagrams (Fig. 1), but for other types of characteristics, solutions could also be derived from these results.

In efficiency scaling-up a widely accepted method which follows from the relationships suggested by the IEC Test Code[6] is to disregard the displacement in n_{11} and Q_{11}. In determining the corresponding points, this means:

method I: $n''_{11}=n'_{11},\ Q''_{11}=Q'_{11}$ ... (1)

Another method is preferred, however, by Nechleba[8,9], Smirnov, and Chistakov also referred to by Hutton[10]:

method II: $\dfrac{n''_{11}}{n'_{11}}=\left(\dfrac{\eta''}{\eta'}\right)^{0.5},\ \dfrac{Q''_{11}}{Q'_{11}}=\left(\dfrac{\eta''}{\eta'}\right)^{0.5}$ ... (2)

Szabó applies this method with a modification taking into consideration in the expression Q_{11} the volumetric efficiency as well[12] which is of some importance in case of Francis turbines. A generalization of the above formulae of Q_{11} was suggested by Vazeille[11]. The calculation of corresponding points can be performed by various further methods, yet the two techniques described above seem to be those most frequently used.

It is easy to realize that methods I and II result in different efficiencies for the prototype, as the full-scale efficiency is calculated respectively from different model efficiencies (Fig. 1). A comparison is shown in Fig. 2 for a tubular turbine prepared from the data given by Osterwalder[2]. At best efficiency, the results in each case show only an insignificant difference. This may be the reason why the problem of corresponding points did not receive sufficient attention earlier.

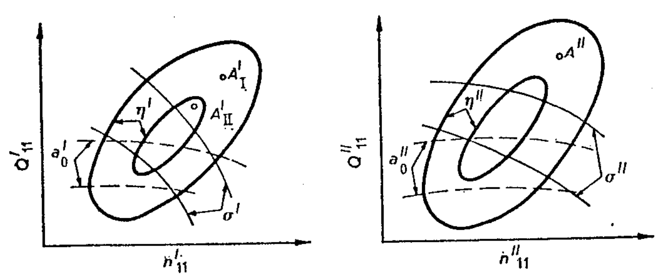

Fig. 1

Notations

A = Point on the efficiency diagrams, Fig. (1)
C_L, C_D = Effective lift, and drag coefficient, of the blade section
D = Runner tip diameter
F_u = Peripheral force coefficient
g = Acceleration due to gravity
H = Net head across the turbine
H_s = Geometric suction head above the tailwater
K = Constant (K = 6·28/60 s/min)
M = Hydraulic torque (for the prototype M = the sum of shaft torque and bearing friction, for the model M = the torque measured by the double-bearing brake)
M_{11} = Unit torque ($M_{11}=M/HD^3$)
m, m_n, m_Q = Exponents
n = Runner speed
n_{11} = Unit speed ($n_{11}=nDH^{-0.5}$)
n_* = Exponent in the Hutton formulae suggested by Osterwalder
p_v = Saturated vapour pressure
p_b = Absolute pressure at suction side (for the prototype p_b = atmospheric pressure, for the model p_b = reduced pressure)
Re = Reynolds number of turbine ($Re=D(2gH)^{0.5}\nu^{-1}$)
Q = Volumetric flow
Q_{11} = Unit flow ($Q_{11}=QD^{-2}H^{-0.5}$)
Q_{11*} = Unit flow at optimum efficiency
u = Blade rotational velocity
v = Absolute fluid velocity
v_r = Relative fluid velocity
V = Assessable friction loss coefficient, ie. that applicable to scale-effect calculation
V_R = Assessable friction-loss coefficient of the runner
x = Discharge ratio ($x=Q_{11}/Q_{11*}$)
β = Angle between relative velocity and the plane of rotation
γ = Specific weight
δ = Dimensionless loss through turbine ($\delta=1-\eta$)
η = Hydraulic efficiency ($\eta=\mathrm{K}Mn/\gamma QH$)
ρ = Fluid density
σ = Thoma cavitation number ($\sigma=1/H[(p_b/\gamma)-H_s-(p_v/\gamma)]$)

Superiors: ′ refers to model, ″ indicates the prototype or another model, quantities without a superior apply to any member of the turbine family.

Further away from the optimum, however, the difference may be higher than the required accuracy of efficiency scaling-up (which is[2] about $\pm 0{\cdot}1\%$) and therefore it may not be neglected any more.

When determining the corresponding points, operating conditions of the turbines concerned are intended to be represented by those points for which the dynamic similarity of flows in turbines is satisfied[13]. In most cases,

* Mechanical Engineer, Department of Hydraulic Machinery, Budapest Technical University; and Hydraulic Laboratory, Ganz-Mávag Co, Budapest, Hungary.

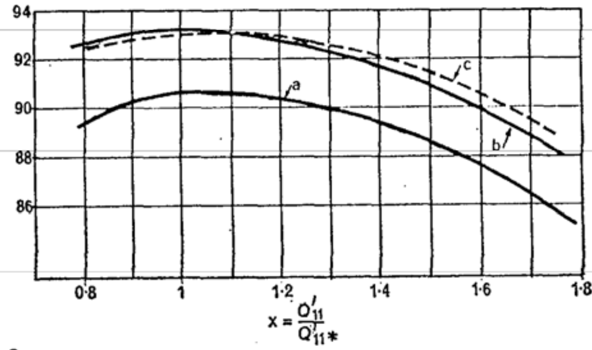

Fig. 2

however, the geometric similarity of flow boundaries does not extend to roughness and clearance formation, the Reynolds number differs and consequently there are no operating conditions exhibiting strict dynamic similarity.

Similarity is thus only approximatively achieved, which means in turn that different principles could be followed in the approximation. This is the reason why different methods exist for the calculation of corresponding points.

In deciding which of the methods should be applied in a given case, the principles on which the methods are based offer guidance for reasonable decisions. In order to derive such principles for methods I and II, let two turbines be considered, satisfying the geometric similarity requirements given in the IEC Test Code[6].

The operating conditions represented by corresponding points on the efficiency diagrams are called "corresponding operating conditions". Whichever method is used for the calculation of corresponding points, in order to ensure geometric similarity as far as possible, the runner-blade angle should be the same for both turbines under corresponding operating conditions.

Other relations for the corresponding operating conditions determined by methods I and II, obtained by the substitution of the definitive equations of n_{11}, Q_{11}, and η into Eqs. (1) and (2) are:

$$\text{method I: } \frac{Q''/D''^2}{n''D''}=\frac{Q'/D'^2}{n'D'},\ \frac{H''}{(n''D'')^2}=\frac{H'}{(n'D')^2} \quad \dots \quad (3)$$

$$\text{method II: } \frac{Q''/D''^2}{n''D''}=\frac{Q'/D'^2}{n'D'},\ \frac{M''}{(n''D'')^2}=\frac{M'}{(n'D')^2} \quad \dots \quad (4)$$

On the basis of these equations, the methods can be characterized as follows: quantities Q/D^2 and nD are of velocity dimension; Q/D^2 is proportional to the axial component of the average velocity of the fluid at the runner; and nD is proportional to the blade rotational velocities. The first equation required by both methods provides for the equality of these velocity proportions.

The difference between the two methods appears in the second requirement: according to method I, H is proportional to the square of velocity Q/D^2 or nD, whereas according to method II, M is proportional to the square of the same velocities. The importance of these proportions can be readily realized in the special case when:

$$D''=D',\ n''=n' \quad \dots \quad (5)$$

apply to the turbine operating under corresponding operating conditions.

In practice these conditions are satisfied, for example, when the Reynolds-number effect is tested with the same model turbine using fluids of different viscosities[2]. The scale effect due to relative-roughness variations can also be tested by making certain surfaces of the model turbine rougher, and conducting efficiency measurements with the same turbine at the same speed.

With Eq. (5) satisfied, Eqs. (3) and (4) reduce to:

$$\text{method I: } Q''=Q',\ H''=H' \quad \dots \quad (6)$$

$$\text{method II: } Q''=Q',\ M''=M' \quad \dots \quad (7)$$

Thus, in this special case both methods require equal flows, method I requires the equality of heads, and method II requires equality of torque. If the efficiencies are unequal due to scale effects then, since the equality of the other variables is ensured, in the case of method I the torque values, and in case of method II the heads will be different.

Knowing these facts, it can be decided in some cases which method leads to a better approximation of dynamic similarity. Studying, for example, the scale effect due to the variations in the relative roughness of the draft tube with the other parameters kept unchanged, the following consideration may be decisive.

According to the usual approximation in hydraulics, the flows in the individual parts of the turbine are considered as independent. If, consequently, the reaction of the draft-tube flow conditions on the flow around the runner is neglected, then, assuming equal diameter, speed and volumetric flow, the identical flow conditions at the runner are ensured by the equality of the torques, while the draft tube efficiency variations will be manifested by the change in the head.

These features are characteristic of method II, and so in testing draft-tube roughness effects the use of method II can be suggested.

If the relative roughness of the guide vanes or spiral casing is varied, similar considerations lead to method II again. However, if the roughness of the runner blade is modified, it would be reasonable to assume that the variations are reflected mainly by the torques, and only to a limited extent by the head, so method I might represent a better approximation.

The problem is much more complex if the Reynolds number varies, since in this case the flow will vary in each part of the turbine. With the diameter, speed and volumetric flow being equal, it is reasonable to assume that, depending on the effect of Reynolds-number variations the shear stress over the runner blades will change as reflected by the torques, whereas the variation of the hydraulic losses will be manifested in the head. Consequently, in this case a third method is required taking into consideration both head and torque variations.

The importance of scale-effects due to Reynolds-number variations is reflected by the fact that it is taken into consideration in most efficiency scale formulae, whereas the effect of the relative roughness, for example, is generally neglected[2].

This article has been written in an attempt to provide a method adaptable in cases characterized by the variation of the Reynolds number for the calculation of the corresponding points. Some general concepts and formulae of the scale-effect theory are used to derive special formulae which finally lead to the expressions of method III suitable for practical calculation.

Scale effects

One of the most important discoveries in the theory of hydraulic machines was the determination of the similarity laws whereupon the characteristics n_{11} and Q_{11} were then introduced. During the initial stage of development, the n_{11}-Q_{11}-η diagrams were considered applicable to the entire family of geometrically similar turbines for all heads.

At this stage of approximation, the corresponding operating conditions of geometrically similar turbines may be characterized by the fact that all the characteristics referred to unity head and diameter are constants:

$$n_{11}=\text{const},\ Q_{11}=\text{const},\ M_{11}=\text{const},\ \eta=\text{const} \quad \dots \quad (8)$$

if the head and diameter vary.

The recognition that the efficiency of larger turbines is generally higher has led to efficiency scaling up. In this stage of approximation, the n_{11}-Q_{11}-η diagrams apply only to the cases of a given diameter and head, respectively, and for another head or diameter they must be scaled up.

Efficiency changes, however, necessarily result in variation of the other characteristic quantities as well. For example, the following relationship holds good:

$$\eta=\mathrm{K}(M_{11}n_{11}/\gamma Q_{11}) \quad \dots \quad (9)$$

where K is constant and therefore if γ is unchanged the efficiency variation will lead to a simultaneous change in at least one of the quantities n_{11}, Q_{11} and M_{11}.

Articles concerning scale effects often present a definition of the scale effect on the efficiency in such a way that the problem represented by the variation of the last-named characteristics is left unsolved. From the practical aspect, these definitions may be regarded as deficient since (as shown above) the efficiency of the prototype cannot be calculated unequivocally without the determination of the n_{11} and Q_{11} variations. Therefore when defining scale effects, it is reasonable to consider all characteristics.

In conformity with the traditional aspects of the turbine scale-effect theory, the laws expressed in Eq. (8) are accepted here as a basis for the scale-effect calculations. The scale effects are the deviations from these laws, which means that the variation of any quantity with reference to unity head and diameter represents a scale effect. The numerical expressions of the scale effects are called scale formulae, thus Eq. (2) gives formulae for the n_{11} and Q_{11} scale effects.

By such an interpretation of the scale effects, the calculation of corresponding points requires two scale formulae—those of n_{11} and Q_{11}—and for unequivocal efficiency scaling-up three scale formulae are needed: for n_{11}, Q_{11}, and η.

The series of characteristics for which scale effects are defined can be completed with other variables as well. When the Thoma cavitation number σ is included the fundamental law of cavitation scale-effect theory is the invariability of sigma in conformity with the above definition. This is the well-known similarity law introduced by Thoma.

The cavitation scale effects are actually the variations of sigma, as interpreted by Holl and Wislicenus[14] among others. According to Nechleba, sigma will also reflect[1,8,9] a scale effect simultaneously with the efficiency variations. For unequivocal cavitation scaling-up, the scale formulae of n_{11}, Q_{11} and σ are required.

Dimensionless characteristics and gravity scale effects

The interpretation of scale effects based on the classical approach described above is completely suitable for practical use, but nevertheless it is reasonable to define the scale effects for a much wider class of variables. The n_{11} and Q_{11} characteristics are dimensional, and there is a widely-spreading trend in hydraulic machinery engineering to use dimensionless variables[5]. A scale-effect theory based on such variables has also been presented by Csanady[13].

The main characteristics of operating condition used by him are: $D, \rho, n, Q, M, E=gH$, and $E_s=(p_b/\rho)-gH_s-(p_v/\rho)$ (net suction energy of turbine). A number of dimensionless variables can be composed of these main characteristics, for example:

$$\frac{nD}{E^{0.5}},\ \frac{Q}{D^2E^{0.5}},\ \frac{M}{\rho ED^3},\ \mathrm{K}\frac{Mn}{\rho EQ},\ \frac{E_s}{E},\ \frac{nQ^{0.5}}{E^{0.75}},\ \frac{Q}{D^3n},\ \frac{nQ^{0.5}}{E_s^{0.75}},\ \dots$$

Based on dimensional analysis, scale effects could be defined as the changes in these dimensionless variables. However theoretically well-established, this definition does not refer directly to the variables used in practice, but by a slight extension this difficulty can be removed.

With the definitive equations of E and E_s taken into consideration, this row of dimensionless variables reveals the n_{11}, Q_{11}, M_{11}, η, σ, ... characteristics as multiplied with various powers of acceleration due to gravity. Such variables are called hereafter variables "derived" from dimensionless variables. Defining scale effects as the changes in the dimensionless characteristics and in those derived therefrom, this definition covers both the classical approach and Csanady's theory.

If the acceleration due to gravity is considered constant, there is no essential difference in the use of the dimensionless and the derived variables. If the value of g varies, then the dimensionless characteristics should be constant, while the derived variables show scale effects according to the power of g in their formulae when expressed by dimensionless variables.

For example, for the n_{11} and Q_{11} variables:

$$\frac{n''_{11}}{n'_{11}}=\left(\frac{g''}{g'}\right)^{0.5},\ \frac{Q''_{11}}{Q'_{11}}=\left(\frac{g''}{g'}\right)^{0.5}$$

These are the formulae of the n_{11} and Q_{11} scale effects due to gravity variations. The changes in the acceleration due to gravity are usually small, and thus this scale effect is less important.

Scale effects due to Reynolds-number changes

If nothing but the Reynolds number of the turbine varies, ie. if the other scale effects are not present or are neglected, and the scaling-up is considered at a given point n'_{11}, Q'_{11} of the efficiency diagram of the model turbine, then the three scale formulae of efficiency scaling-up are of the form:

$$n_{11}=\mathrm{f}_n(Re),\ Q_{11}=\mathrm{f}_Q(Re),\ \eta=\mathrm{f}_\eta(Re) \quad \dots \quad (10)$$

Function f_η will not be dealt with here but regarded as known, but it is assumed that f_η is increasing as the Reynolds number increases. This applies to all efficiencies obtained from the known efficiency scale formulae.

Thus it follows that there exists an inverse function of f_η and using this, n_{11} and Q_{11} can be expressed by η:

$$n_{11}=\mathrm{g}_n(\eta),\ Q_{11}=\mathrm{g}_Q(\eta) \quad \dots \quad (11)$$

Applying these functions to a couple of turbines:

$$\frac{n''_{11}}{n'_{11}}=\frac{\mathrm{g}_n(\eta'')}{\mathrm{g}_n(\eta')},\ \frac{Q''_{11}}{Q'_{11}}=\frac{\mathrm{g}_Q(\eta'')}{\mathrm{g}_Q(\eta')} \quad \dots \quad (12)$$

These equations reveal the general form of the basic equations of methods I and II. Thus it is verified here that if nothing but the Reynolds-number variation is reckoned with, then the basic formulae in the calculation of corresponding points can always be written as a function of η.

Eq. (12) can be simplified by approximation. Assuming that g_n and g_Q can be expanded into power series:

$$n''_{11}-n'_{11}=\frac{\mathrm{dg}_n}{\mathrm{d}\eta}(\eta''-\eta')+\dots,$$

$$Q''_{11}-Q'_{11}=\frac{\mathrm{dg}_Q}{\mathrm{d}\eta}(\eta''-\eta')+\dots \quad \dots \quad (13)$$

Since scale effects are usually small, the square and higher powers of $(\eta''-\eta')$ can be neglected in the first approximation, and they are already omitted in Eq. (13). Introducing the quantities:

$$m_n=\frac{\mathrm{d}g_n}{\mathrm{d}\eta}\frac{\eta'}{n'_{11}},\ m_Q=\frac{\mathrm{d}g_Q}{\mathrm{d}\eta}\frac{\eta'}{Q'_{11}} \qquad \ldots\ (14)$$

leads to the following form of Eq. (12):

$$\frac{n''_{11}-n'_{11}}{n'_{11}}=m_n\frac{\eta''-\eta'}{\eta'},\ \frac{Q''_{11}-Q'_{11}}{Q'_{11}}=m_Q\frac{\eta''-\eta'}{\eta'} \qquad \ldots\ (15)$$

which are the first approximations of the power functions:

$$\frac{n''_{11}}{n'_{11}}=\left(\frac{\eta''}{\eta'}\right)^{m_n},\ \frac{Q''_{11}}{Q'_{11}}=\left(\frac{\eta''}{\eta'}\right)^{m_Q} \qquad \ldots\ (16)$$

The derivation of these formulae does not involve any hydraulic assumption and thus they may be considered as the general forms for the calculation of corresponding points in the case of Reynolds-number variations. The formulae of methods I and II are the special cases of Eq. (16) with $m_n=m_Q=0$ and $m_n=m_Q=0{\cdot}5$, respectively.

In general, the exponents m_n and m_Q may differ for the different turbine families and depend on the point of the n'_{11}-Q'_{11}-η' diagram at which the corresponding point was determined. Thus, in case of a given turbine family and a given point, for the calculation of the corresponding points two constants, m_n and m_Q have to be known.

Exponents m_n and m_Q for Kaplan turbines

The m_n and m_Q exponents can be calculated for Kaplan turbines conveniently in the special case expressed in Eq. (5), ie. when considering two turbines of identical diameter and speed with only their Reynolds numbers being different. From the practical aspect this may be considered as operating the same turbine with a different viscosity fluid, eg. Osterwalder's experiments[2].

The first assumption of hydraulic character accepted here for the corresponding operating conditions is:

$$Q''=Q' \qquad \ldots\ (17)$$

Both methods I and II have also applied this assumption. It follows from this, using also Eq. (5) and (16), that:

$$m_n=m_Q=m \qquad \ldots\ (18)$$

from which, in turn, the formulae adaptable for the calculation of corresponding points are:

$$\frac{n''_{11}}{n'_{11}}=\left(\frac{\eta''}{\eta'}\right)^{m},\ \frac{Q''_{11}}{Q'_{11}}=\left(\frac{\eta''}{\eta'}\right)^{m} \qquad \ldots\ (19)$$

Exponent m is in close connection to the head and torque variations. Taking into consideration that the diameter, speed, and volumetric flow is identical for both cases, the following equations apply to the first approximation:

$$\frac{n''_{11}-n'_{11}}{n'_{11}}=-\frac{1}{2}\frac{H''-H'}{H'} \qquad \ldots\ (20)$$

$$\frac{\eta''-\eta'}{\eta'}=\frac{M''-M'}{M'}-\frac{H''-H'}{H'} \qquad \ldots\ (21)$$

and by making use of Eq. (15):

$$\frac{H''-H'}{H'}=(-2m)\frac{\eta''-\eta'}{\eta'},$$

$$\frac{M''-M'}{M'}=(1-2m)\frac{\eta''-\eta'}{\eta'} \qquad \ldots\ (22)$$

According to Eq. (21) the relative efficiency variation consists of two parts: the relative changes in head and torque, respectively. Their ratio is shown by Eq. (22): a part of $2m$ is due to head variation, and a part of $(1-2m)$ can be attributed to torque change. This actually illustrates the meaning of exponent m.

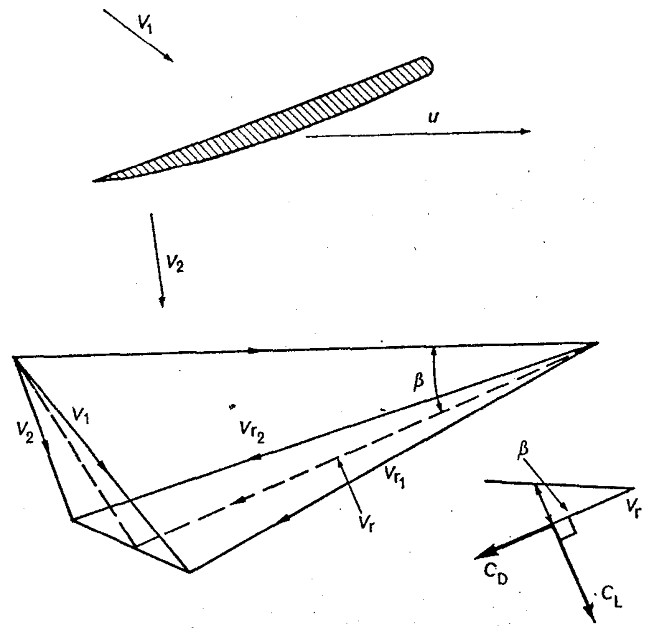

Fig. 3

The second assumption accepted in order to make the calculation of corresponding points definite refers to the velocity triangles. The runner blade velocity diagram of Hutton's paper[1] is shown in Fig. 3, which represents the average conditions on the blading. The incidence angle of the average relative velocity is assumed to be the same for the two turbines under corresponding operating conditions:

$$\beta''=\beta'=\beta \qquad \ldots\ (23)$$

This assumption may be characterized by its providing with the previous assumptions involving equal diameter, speed, and flow, for identical average runner velocity triangles.

Now the torque variations will be estimated on the basis of our earlier assumptions. According to Fig. 3 the *peripheral force coefficient* for either of the two turbines is:

$$F_u=C_L\sin\beta-C_D\cos\beta \qquad \ldots\ (24)$$

and therefore:

$$\frac{M''-M'}{M'}=\frac{F_u''-F_u'}{F_u'}=\frac{(C_L''-C_L')\sin\beta-(C_D''-C_D')\cos\beta}{C_L\sin\beta-C_D'\cos\beta} \qquad \ldots\ (25)$$

According to the cascade theory the variation of the Reynolds number has, *in the flow around the blades* under invariable velocity at infinity, a number of different consequences: variation of the boundary layer thickness leads to changes in the flow velocity outside of the boundary layer, and at the trailing edges of the profiles the theoretical rear stagnation point is displaced[13].

Although these effects ought to be analysed in detail, in scale-effect calculations they are usually neglected[1,2], and the flow outside the boundary layer is considered as invariable. In conformity to this assumption:

$$C_L''=C_L' \qquad \ldots\ (26)$$

is accepted. Factor C_D can be expressed[1] with the runner losses δ_R:

$$C_D = C_L \delta_R \sin\beta\cos\beta \qquad \dots (27)$$

and for the loss variations[1,2]:

$$\frac{\delta_R''}{\delta_R'} = 1 - V_R\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n_*}}\right],$$

$$\frac{\delta''}{\delta'} = 1 - V\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n_*}}\right] \qquad \dots (28)$$

A variable exponent n_* is used here according to a suggestion by Osterwalder[2]. By a simple manipulation of Eqs. (22), (25), (26), (27) and (28), one can obtain:

$$m = \frac{1}{2}\left(1 - \frac{\eta'\cos^2\beta}{1 - \delta_R'\cos^2\beta}\,\frac{\delta_R' V_R}{\delta V}\right) \qquad \dots (29)$$

The quantities contained in this expression are given in Hutton's paper in the function of the discharge ratio $x = Q'/Q'_*$, where Q'_* pertains to optimum efficiency.

Assuming $\cos\beta = 0{\cdot}96$ and using an average efficiency curve for the model shown in Fig. 4, the value of m has been calculated from Hutton's original data (Ref. [1], Figs. 5, 6, 7), from Fauconnet's results (Ref. [1] discussion), and from the loss proportions given by Osterwalder (Ref. [2], Fig. 6). The results are shown in Fig. 4.

The character of the curves thus plotted is identical, their differences may be attributed to the fact (among other factors) that different turbine families were tested by each of the authors. A mean value can be obtained, however, by making use of the straight line plotted in the same drawing whereby the exponent m can be estimated for any Kaplan turbine family.

Accordingly, the fundamental formulae of the new method are:

method III:

$$\frac{n''_{11}}{n'_{11}} = \left(\frac{\eta''}{\eta'}\right)^m,\ \frac{Q''_{11}}{Q'_{11}} = \left(\frac{\eta''}{\eta'}\right)^m,\ m = 0{\cdot}15x,\ x = \frac{Q'_{11}}{Q'_{11*}} \qquad \dots (30)$$

Some numerical values deserve mention. At the point of best efficiency ($x=1$), Eq. (30) gives $m = 0{\cdot}15$ which means—by Eq. (25)—that 30% of the efficiency variation is due to head variation and 70% to torque change. However, in the case of large discharge ratio, at $x=2$ for example, the opposite situation exists as now $m = 0{\cdot}3$ and therefore 60% will be due to head, and 40% to torque.

Application

In the case of Kaplan turbines, efficiency scaling-up is best performed—according to the above analysis—by using method III for the calculation of corresponding points. Although the Hutton-type efficiency-scale formulae were employed earlier[6] with method I, their use may also be suggested for method III, as it seems that the basic assumptions accepted for the derivation of these efficiency-scale formulae[1,2] approximate much closer to the assumptions adopted in the derivation of method III than the relationships manifested in the application of method I.

However, the difference between the results obtained by the two methods is negligible in the case of smaller discharge ratios.

In the case of Francis turbines, no suitable loss analysis is available for the determination of exponents m_n and m_Q as yet.

The cavitation scale-effect due to Reynolds-number variations can be calculated for Kaplan turbines by the Nechleba formula[9] applied together with method II. Cavitation scale-effect calculations, however, are based again on the assumption that the flow conditions outside the boundary layer do not vary. For this reason it is much better to use method III for the calculation of corresponding points.

Following Nechleba's line of reasoning[9] reveals that the equations of method II were employed only in the last step. Using an optional exponent, the Nechleba equation has the following form:

$$\frac{\sigma''}{\sigma'} = \left(\frac{Q''_{11}}{Q'_{11}}\right)^2 = \left(\frac{\eta''}{\eta'}\right)^{2m} \qquad \dots (31)$$

This is the cavitation scale formula based on the Nechleba theory applicable for Kaplan turbines together with method III.

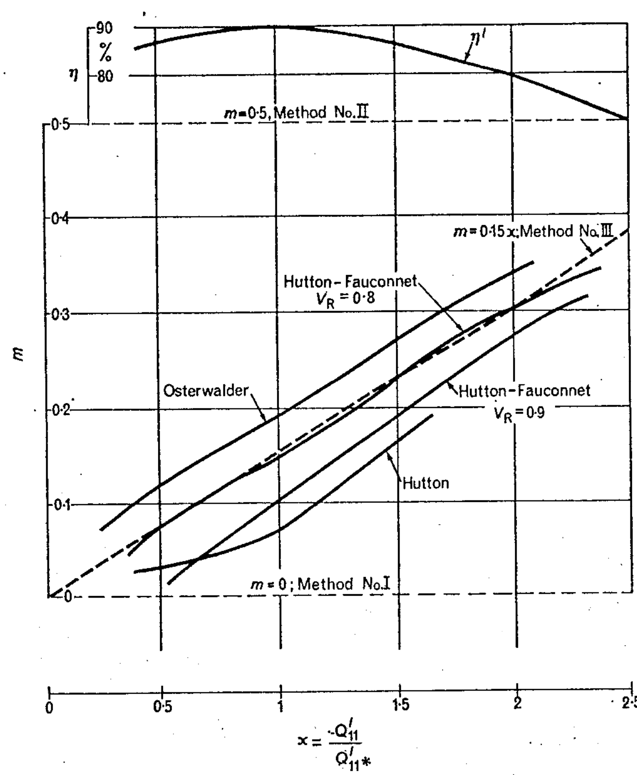

Fig. 4

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