ANALYSIS OF P.I.D. GOVERNORS IN MULTIMACHINE SYSTEM

N. S. Dhillon
Manitoba Hydro
Winnipeg, Manitoba
Canada

H. E. Wichterl

ABSTRACT
The effect of derivative gain and other governor parameters on the stability of a single machine supplying an isolated load is investigated. The effect of derivative gain and other parameters of the governors in two plant systems is investigated. The plants have different types of governors and are feeding an HVDC system. State space method is used for analysis and results are compared with detailed nonlinear model CSMP simulations.

INTRODUCTION
At present the Manitoba HVD system is being supplied by a single powerhouse at Kettle. The plant has 12 hydro-electric generating units each rated at 120 MVA, 1.05 P.F. A more detailed description is given in References [1] and [2].

Kettle is the first of up to five plants to be built on the Nelson River. The second plant being developed is Long Spruce. The first unit of this plant will be commissioned in the summer of 1977. The plant, when fully developed, will have 10 units, each rated at 115 MVA, 0.85 P.F. The plant is connected to the Radisson rectifier bus by three 230 kV lines and three 138/230 kV autotransformers. A single line diagram is shown in Figure 1.

The governors at Kettle are of electro-hydraulic P.I.D. Type. The governors have a permanent droop gate feedback, but no temporary droop feedback. The block diagram is shown in Figure 2. The constants Kd, Kp and Kj represent the derivative, proportional and integral gains respectively.

Fig. 1. Single line diagram of Kettle, Radisson and Long Spruce System.

The governors at Long Spruce will also be of electro-hydraulic type. The block diagram is shown in Figure 3. The governors have temporary droop feedback signal of the gate position and permanent droop feedback signal from electrical power output of the machine.

A substantial amount of work [3], [4], [5] has been done toward defining the stability region of a hydraulic turbine governor control system. In a recent paper [6] Thorne and Hill used state space methods to study the stability region of a hydraulic turbine generating unit having a P.I.D. governor. Their work shows the stability boundaries as a function of proportional and integral gains but no reference is made to the derivative gain.

As Kettle and Long Spruce plants are electrically close to each other, it was felt that the effect of parameter variation on the combined operation be studied. In this paper, the effect of variation of the derivative gain and other parameters, in isolated operation and the combined operation, is investigated. In the combined operation the effect of changes in electrical powers due to the changes in rotor angles is included. No attempt is made to define stability boundaries.

State space method is used for the analysis. First, state variables are defined for the system to be studied. The equations are then written using standard matrix form of:

\[ \dot{X} = AX \]

where the dot over the X implies the first derivative with respect to time and A is a matrix whose coefficients depend on the parameters of the system being considered. Finally, the eigenvalues of the system are determined to see if the real part of any eigenvalue is positive, which indicates instability. The results are compared with a CSMP [7] simulation of the nonlinear model of the same system.

ASSUMPTIONS
The following assumptions are made in forming the state variable formulation:
1. a linear system representation is valid, this implies that only small signal disturbances are to be considered,
2. the relationship between mechanical torque and gate is given by:

\[ T_{\text{mech}} = \frac{1 - Dw_{s}}{\frac{1}{2}} \text{Gate} \]

where \( T_{\text{mech}} \) is the mechanical torque, \( w_{s} \) is the angular speed of the turbine, and \( D \) is a constant.
3. all damping torques due to prime mover, generator and the HVDC system are negligible,
4. HVDC system is represented by a constant admittance load, and
5. generators are represented by a constant voltage behind transient reactance.

LONG SPRUCE-ISOLATED OPERATION
The state variables chosen are shown in Figure 3. The equations of the variables in matrix notation are:
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
\frac{-1}{T_1} & \frac{-1}{R_1T_v} & \frac{-1}{R_1T_v} & \frac{-1}{T_1} & \frac{1}{2H_1} & \frac{1}{2H_1} \\
\frac{-1}{R_1T_v} & \frac{-1}{T_v} & \frac{-1}{R_1T_v} & \frac{-4}{R_1T_v} & \frac{-2}{T_w} & \frac{-2}{T_w} \\
1 & \frac{1}{R_3} & \frac{1}{R_3} & \frac{1}{R_3} & \frac{1}{T_1} & \frac{1}{T_1} \\
r^4 & \frac{1}{R^3} & \frac{1}{R^3} & \frac{1}{R^3} & \frac{1}{R^3} & \frac{1}{R^3} \\
\frac{-2}{T_w} & \frac{-2}{T_w} & \frac{-2}{T_w} & \frac{-2}{T_w} & \frac{-2}{T_w} & \frac{-2}{T_w} \\
\frac{1}{2H_1} & \frac{1}{2H_1} & \frac{1}{2H_1} & \frac{1}{2H_1} & \frac{1}{2H_1} & \frac{1}{2H_1}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\]

\[\Delta T_{e1}\]

(1)

Fig. 2. Kettle turbine and governor

Fig. 3. Long Spruce turbine and governor.
This is the standard state variable formulation which in matrix notation is:

\[ \dot{X} = A \cdot X + B \cdot U \]

where \( U \) is the system disturbance, which in this case is the value of electrical load.

From control theory the stability of a system described by the above equation can be determined by examining the sign of the real parts of the eigenvalues of \( A \). The eigenvalues of \( A \) for a set of parameters were calculated using IBM subroutines HSBG and ATEIG[8]. The values of parameters and the eigenvalues are shown in Table 1.

The two pairs of complex conjugate eigenvalues have the dominating effect.

The values of parameters shown in Table 1 were arbitrarily chosen as the base values.

The effect of individual parameters is examined by changing one parameter and keeping others at their base values.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivative Gain R3</td>
<td>0.9</td>
</tr>
<tr>
<td>Temporary Droop r'4</td>
<td>0.3</td>
</tr>
<tr>
<td>H1</td>
<td>4.0</td>
</tr>
<tr>
<td>Reset Time R'3</td>
<td>4.0 Sec.</td>
</tr>
<tr>
<td>R1</td>
<td>0.3</td>
</tr>
<tr>
<td>Twl</td>
<td>1.28</td>
</tr>
<tr>
<td>r4</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**Derivative Gain R3**

Derivative gain was varied from 0.3 to 1.5. Figure 4a shows the loci of the dominating roots. The increase in derivative gain up to about 1.3 increases the damping of \( \lambda 5 \) and \( \lambda 6 \) without changing the frequency appreciably. The locus of \( \lambda 3 \) and \( \lambda 4 \) indicates that very high values of derivative gain R3 will cause these eigenvalues to move to the right side of the imaginary axis.

A CSMP study for 50% load rejection was conducted for different values of R3. Figure 4b shows the frequency variations as obtained from these simulations. It is interesting to note that for \( R3 = 4.0 \) response becomes oscillatory. Although CSMP simulations involved 50% load change (not a small disturbance) yet the results are in good agreement with the eigenvalue analysis.

**Reset Time Constant R'3**

Reset time constant was varied from 3.0 sec. to 7.0 sec. Figure 5a shows the loci of the dominating eigenvalues. The increase in reset time constant increases the damping of \( \lambda 5 \) and \( \lambda 6 \) but the change in oscillation frequency is small.

Figure 5b shows the frequency variations for 50% load rejections as obtained from the CSMP simulations, and the expected damping effect can be seen with the increase in reset time.

**Temporary Droop r'4**

Variation of temporary droop \( r'4 \) between 0.2 and 0.6 was considered. Figure 6a shows the loci of the roots. The increase in \( r'4 \) decreases the frequency of the most dominant eigenvalue. But the increase in \( r'4 \) increases the damping only up to a certain level and a further increase in \( r'4 \) results in a decrease in damping. These observations are verified by the results shown in Figure 6b.

![Fig. 4a. Effect of derivative gain R3 on the eigenvalues of Long Spruce plant.](image)

![Fig. 4b. Frequency variation of Long Spruce plant for different values of derivative gain R3.](image)

Figure 7a shows the plot of \( \lambda 5 \) for a derivative gain (R3) of 0.3 and various values of \( R'3 \) and \( r'4 \). Constant R3 and constant r4 lines are also shown. Figure 7b shows similar curves for a higher value of R3. For a given derivative gain, the values of temporary droop and reset time constant can be chosen to obtain a desired system response. Figures 7a and 7b show that the locus of \( \lambda 5 \) gets shifted to the left for a high derivative gain. This indicates that the stability limit can be increased by increasing derivative gain within the stable range.

**PARALLEL OPERATION OF LONG SPRUCE AND KETTLE**

In the previous section isolated operation of the Long Spruce plant was considered. However, in practice this plant will be operating in parallel with the Kettle plant. The effects of individual parameters of the governors of each plant on system stability are considered in this section.

Details of state variable formulation are given in the Appendix. For purpose of analysis two types of system configurations were considered.
Fig. 5a. Effect of reset time $R^3$ on the eigenvalues of Long Spruce plant.

Fig. 5b. Frequency variation of Long Spruce plant for different values of reset time $R^3$.

Fig. 6a. Effect of temporary droop $r^4$ on the eigenvalues of Long Spruce plant.

Fig. 6b. Effect of temporary droop $r^4$ on the frequency response of Long Spruce plant.

Fig. 7a. Derivative Gain $R^3 = 0.3$.

Fig. 7b. Tracking of dominant eigenvalue as function of $r^4$ and $R^3$. 

Case 1

In this case 10 machines at Kettle and 4 at Long Spruce were assumed to be in operation. All machines are assumed to be fully loaded. This configuration is representative of initial stages of Long Spruce.

Case 2

In this case 4 machines at Kettle and 10 machines at Long Spruce are considered. All machines are assumed to be fully loaded.

It may be noted here that unless specified otherwise, the discussion that will follow refers to Case 1.

Table II shows the governor parameters and the eigenvalues of the combined system. These values are arbitrarily chosen as the base values.

From Table II it can be seen that \( \lambda_{11} \), \( \lambda_{12} \), \( \lambda_{13} \) and \( \lambda_{14} \) are the dominant eigenvalues.

**TABLE II**

PARAMETERS AND EIGENVALUES
COMBINED OPERATION

<table>
<thead>
<tr>
<th>Governor</th>
<th>( r'4 = 0.3 )</th>
<th>( R = 5.0 )</th>
<th>( r_4 = 0.03 )</th>
<th>( H_1 = 6.0 )</th>
<th>( T = 1.28 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kettle Governor</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Derivative Gain</td>
<td>( K_d = 0.75 )</td>
<td>( H_k = 4.0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportional Gain</td>
<td>( K_p = 2.5 )</td>
<td>( T_{wk} = 1.0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Integral Gain</strong></td>
<td>( K_i = 0.5 )</td>
<td>( \sigma = 0.03 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Eigenvalues**

\[
\begin{align*}
\lambda_1 &= -55.74 \\
\lambda_2 &= -19.07 \\
\lambda_3 &= -13.50 \\
\lambda_4 &= -9.73 \\
\lambda_5 &= -2.24 \\
\lambda_6 &= -0.44 \\
\lambda_7 &= -0.015 \\
\lambda_8 &= -0.0 \\
\lambda_9, \lambda_{10} &= -1.36 \pm j0.49 \\
\lambda_{11}, \lambda_{12} &= -0.091 \pm j0.36 \\
\lambda_{13}, \lambda_{14} &= -0.006 \pm j12.17
\end{align*}
\]

Frequency of oscillation corresponding to \( \lambda_{13} \) and \( \lambda_{14} \) is 12.17 rad/sec (1.94 Hz). The damping at this frequency is small. This frequency represents the electromechanical oscillations between the two plants. These oscillations have been observed in stability studies. Both plants will have power stabilizers to damp out these oscillations.

The effect of each parameter is observed by changing one parameter over a practical range and keeping all other parameters constant.

**Derivative Gain (R3)—Long Spruce**

The derivative gain of Long Spruce was varied from 0.3 to 1.0 and the only significant effect was on \( \lambda_{13} \) and \( \lambda_{14} \). Figure 8 shows the locus of \( \lambda_{13} \) for Case 1 and Case 2. Note that as \( R3 \) increases the damping of this mode decreases and becomes negative. Also the effect of change in \( R3 \) is greater for Case 1 and comparatively less for Case 2.

Case 1 was simulated on CSMP. The CSMP model included nonlinearities of the governor. The synchronous machines were represented by a machine model with transient saliency and static exciters were also included. Power stabilizers were not included. D.C. power was suddenly reduced by 25% of the initial value. Figure 8b shows the variation of accelerating power of the Kettle plant for two values of \( R3 \).

Although the system is unstable in both cases yet, the oscillations build up much quicker for a higher value of derivative gain.

**Derivative Gain (Kd)—Kettle**

The derivative gain of Kettle governor was varied from 0.75 to 3.0 and only \( \lambda_{13} \) and \( \lambda_{14} \) were affected. Figure 9 shows the variation of \( \lambda_{13} \) for both cases. The effect of the derivative gain of Kettle is similar to that of Long Spruce.

**Reset Time Constant (R3)—Long Spruce**

Reset time constant was varied from 1.0 to 7.0 and the significant effect was noticed on \( \lambda_{11} \) and \( \lambda_{12} \). Figure 10 shows that an increase in \( R3 \) increases the damping but the effect is more pronounced in 1.0 to 3.0 second range. The change in frequency of oscillation is small. A CSMP study was made to verify the results. In this test case Kettle was represented by 10 m/c and Long Spruce by 4 m/c. In this study power stabilizers were included to eliminate the effect of \( \lambda_{13} \) and \( \lambda_{14} \). Generator and turbine damping was made equal to 0.5 p.u. (on m/c rating).

Figure 13 shows the frequency variations due to 25% load rejection. Curves A and B correspond to \( R3 = 5.0 \) and \( R3 = 2.5 \) respectively. Note that the frequency of oscillations given by curves is approximately 0.35 rad/sec as against 0.36 rad/sec given by the eigenvalue analysis. The effect of change in \( R3 \) is also consistent with that shown in Figure 10.

**Temporary Droop (r'4)—Long Spruce**

The variation in \( r'4 \) has significant effects on \( \lambda_{11} \) and \( \lambda_{12} \) only. Figure 11 shows the locus of \( \lambda_{11} \). The change in \( r'4 \) effects both damping and frequency of oscillation. It is interesting to note that with the increase in temporary droop the damping increases up to a certain level and then starts decreasing. The system response is more sensitive to \( r'4 \) in the 0.1 to 0.3 range than in 0.3 to 0.6 range.

Curves A and C in Figure 13 show the frequency variation for values of \( r'4 \) equal to 0.3 and 0.1 respectively. Note that for smaller value of temporary droop the damping is small. Frequency of oscillation obtained from Curve C is 0.42 rad/sec and from Figure 11 is 0.44 rad/sec.

**Proportional Gain (Kp)—Kettle**

The proportional gain of Kettle was varied from 2.5 to 5.0. It was noted that an increase in proportional gain increased the frequency of oscillation of \( \lambda_{11} \). The effect on damping is not too significant. The locus of \( \lambda_{11} \) are shown in Figure 12.

Curves A and D in Figure 13 show the frequency responses for values of \( Kp \) equal to 2.5 and 5.0 respectively. The frequency of oscillation changes from 0.36 rad/sec to 0.55 rad/sec (Figure 13). This is in good agreement with Figure 12.

**Integral Gain (Ki)—Kettle**

The increase in integral gain from 0.5 to 0.7 changed the real part of \( \lambda_{11} \) and \( \lambda_{12} \) from -0.091 to -0.071. Thus making the response more oscillatory and increasing the settling time.
Fig. 8a. Locus of $\lambda_{13}$ as function of derivative gain $R3$ of Long Spruce governor.

Fig. 8b. Effect of derivative gain $R3$ on the joint operation of Kettle and Long Spruce.

Fig. 9. Effect of derivative gain $Kd$ of Kettle governor.

Fig. 10. Locus of $\lambda_{11}$ as function of reset time constant $R'3$ of Long Spruce governor.

Fig. 11. Effect of temporary droop $r'4$ of Long Spruce governor on $\lambda_{11}$.
CONCLUSION

1. Successful application of the state variable method of analysis, to study the effect of parameter variation on the stability of two hydroelectric plants operating in parallel, has been demonstrated.

2. It has been demonstrated that too high derivative gain can cause the system to go unstable.

3. For the multimachine system studied, the increase in derivative gain decreases the damping of electromechanical oscillations.

4. The effect of variation of reset time constant and temporary droop on the stability of a two machine system is similar to that in isolated operation.

REFERENCES


NOMENCLATURE
R3 Derivative gain of Long Spruce governor
R'3 Reset time constant of Long Spruce governor
r 4 Long Spruce governor steady state speed droop
r'4 - Long Spruce governor temporary speed droop
Kd Kettle governor derivative gain
Kp Kettle governor proportional gain
Ki Kettle governor integral gain
σ Kettle governor steady state speed droop
R1 Long Spruce servomotor time constant
Tv Distributing valve time constant
Tp Kettle governor pilot servo time constant
Tw1 Long Spruce water starting time
Twk Kettle water starting time
HI Inertia constant of Long Spruce synchronous machine
Hk Inertia constant of Kettle synchronous machine
ia, ib Real and imaginary components of machine currents respectively
va, vb Real and imaginary components of machine terminal voltages respectively
E' Voltage behind transient reactance
Ea, Eb Real and imaginary components of the voltage behind transient reactance respectively
x'd Transient reactance
Δ Rotator angle
Δ = [Δ1, Δ2]
I [ia1, ib1, ia2, ib2]
V [va1, vb1, va2, vb2]

APPENDIX
STATE VARIABLE FORMULATION
KETTLE–LONG SPRUCE SYSTEM

Electrical Torque

By eliminating the Radisson bus after the load flow study and retaining only the terminal buses of generators, the currents can be calculated as follows:

\[
\begin{bmatrix}
ia_1 \\
ib_1 \\
ia_2 \\
ib_2
\end{bmatrix}
= [Y] \times
\begin{bmatrix}
va_1 \\
vb_1 \\
v_a \\
v_b
\end{bmatrix}
\quad (2)
\]

Where subscripts 1 and 2 refer to Long Spruce and Kettle respectively and \( Y \) is the network admittance matrix in the real form, the incremental form of equation (2) can be written as:

\[
\Delta i = Y \Delta V
\quad (3)
\]

From the vector diagram, we have

\[
va = E' \cos \delta + ibx'd
\]
\[
vb = E' \sin \delta - ia.x'd
\]

Writing similar equations for both machines and putting in the matrix form, we have

\[
\begin{bmatrix}
\Delta V
\end{bmatrix} =
\begin{bmatrix}
-Eb_1 \\
Ea_1 \\
-Eb_2 \\
Ea_2
\end{bmatrix}
\begin{bmatrix}
[\Delta \delta_1] \\
[\Delta \delta_2]
\end{bmatrix}
+ \begin{bmatrix}
x'd_1 \\
x'd_2
\end{bmatrix}
\begin{bmatrix}
\Delta l
\end{bmatrix}
\quad \text{or}
\]

\[
\Delta V = \text{ET.} \Delta \delta - \text{XM.} \Delta l
\quad (4)
\]

Eliminating \( \Delta V \) from equations (2) and (4) we have

\[
\Delta i = \text{BE.} \Delta \delta
\quad (5)
\]

where matrix \( \text{BE} = [1. + Y.XM] \times \text{Y.ET} \)

The incremental electrical torques are given by:

\[
\begin{bmatrix}
\Delta T_1 \\
\Delta T_2
\end{bmatrix} =
\begin{bmatrix}
[ia_1] & [ib_1] \\
[ia_2] & [ib_2]
\end{bmatrix}
\begin{bmatrix}
Z\Delta V + [va_1] & [vb_1] \\
va_2 & vb_2
\end{bmatrix}
\quad \text{or}
\]

\[
\Delta T = \text{IR.} \Delta \delta + \text{VE.} \Delta l
\quad (6)
\]

where \( \text{IR} \) and \( \text{VE} \) are the matrices of equation (6).

From equations (5) and (6a) we have

\[
\Delta T = \text{PK.} \Delta \delta
\quad (7)
\]

where matrix \( \text{PK} = \text{IR.ET} - \text{IR.XM.BE + VE.BE} \)

Turbine Power

The state variable form of Long Spruce plant is given by equation (1).

State variable form of Kettle plant can be written as

\[
Z = A2Z + B2. \Delta T_2
\quad (8)
\]

where

\[
A2 =
\begin{bmatrix}
1 & -K_i \\
\frac{1}{T_p} & \frac{1}{T_p} & -K_{d, \sigma} & -K_{p, \sigma} & -K_d & -K_p \\
\frac{1}{T_v} & -\frac{1}{T_v} & \frac{1}{T_v} & \frac{1}{T_v}
\end{bmatrix}
\]

B2 = \begin{bmatrix} 0, 0, a, 0, \frac{1}{2HK}, -\frac{2}{Twk} \end{bmatrix}

Rotor Angles

\[
p \Delta \delta_1 = 377. X_6
\quad (9)
\]
\[
p \Delta \delta_2 = 377. Z_6
\quad (10)
\]

where \( X_6 \) and \( Z_6 \) are the frequency deviations of Long Spruce and Kettle generators respectively.

Equations (1), (7), (8), (9) and (10) are then combined and rearranged to obtain matrix \( A \).